Graphical Abstract

Improvements Proposed to Noisy-OR Derivatives for Multi-Causal Analysis: A Case Study of Simultaneous Electromagnetic Disturbances

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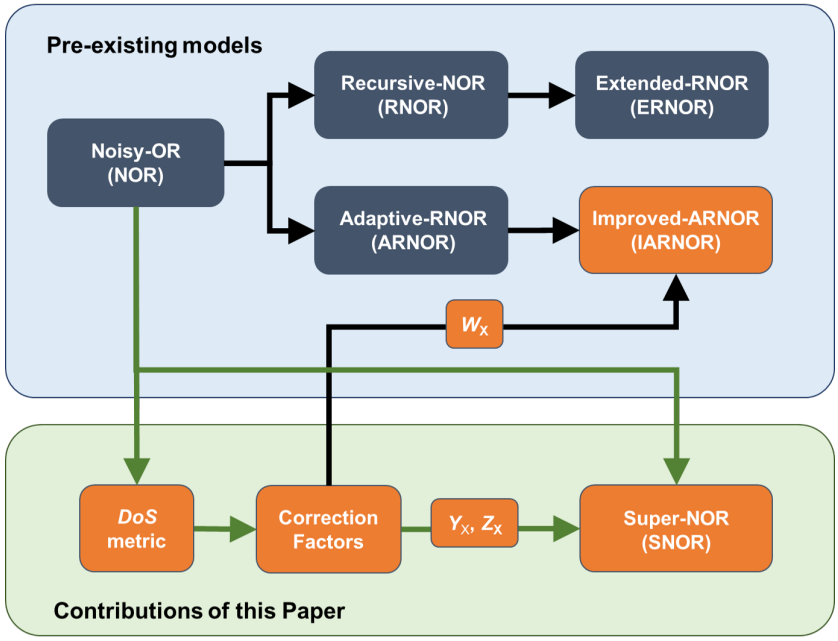

Highlights

Improvements Proposed to Noisy-OR Derivatives for Multi-Causal Analysis: A Case Study of Simultaneous Electromagnetic Disturbances

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- The noisy-OR (NOR) model and various derivatives that are available in the literature, including recursive-NOR (RNOR), adaptive RNOR (ARNOR), improved ARNOR (IARNOR), extended RNOR (ERNOR), are evaluated for the prediction of integrated circuit failures due to electromagnetic interference at up to 10 simultaneous frequencies.
- A new model called super-NOR (SNOR) is proposed to eliminate the limitations of the existing NOR derivatives (i.e., invalidity, inconsistency, or both).
- Given inter-causal dependence information, the multi-causal effect prediction accuracy of the SNOR model is found to be superior to the NOR and its existing derivatives.

Improvements Proposed to Noisy-OR Derivatives for Multi-Causal Analysis: A Case Study of Simultaneous Electromagnetic Disturbances

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Abstract

In multi-causal analysis, the *independence of causal influence* (ICI) assumed by the *noisy-OR* (NOR) model can be used to predict the probability of the effect when several causes are present simultaneously, and to identify (when it fails) *inter-causal dependence* (ICD) between them. The latter is possible only if the probability of observing the multi-causal effect is available for comparison with a corresponding NOR estimate. Using electromagnetic interference in an integrated circuit as a case study, the data corresponding to the probabilities of observing failures (effect) due to the injection of individual (single cause) and simultaneous electromagnetic disturbances having different frequencies (multiple causes) were collected. This data is initially used to evaluate the NOR model and its existing derivatives, which have been proposed to reduce the error in predictions for higher-order multi-causal interactions that make use of the available information on lower-order interactions. Then, to address the identified limitations of the NOR and its existing derivatives, a new deterministic model called *Super-NOR* is proposed, which is based on correction factors estimated from the available ICD information.

Keywords: Noisy-OR (NOR), inter-causal dependence, Bayesian networks, electromagnetic interference (EMI), multi-frequency disturbances,

integrated circuits (ICs).

1. Introduction

Bayesian networks (BN) are directed acyclic graphs that have been used in many domains for modelling a set of variables and their conditional dependencies. One such application of BN is to capture the relationship between multiple causes and a common effect. The fundamental limitation of using a discrete BN for multi-causal analysis is the exponential growth of conditional probability table (CPT) entries that are required for the effect node as the number of causal nodes rises. Nevertheless, using the popular *noisy-OR* (NOR) model introduced in [1, 2], this exponential growth can be reduced to a linear problem. The NOR model assumes *independence of causal influence* (ICI) [3], i.e., simultaneously occurring causes contribute independently to produce the effect, hence the probability values of each individual cause leading to the effect are sufficient to estimate the probability of observing the effect for higher-order combinations of these causes. Applications of the NOR model to make such predictions can be found in [4, 5, 6, 7].

However, when there is *inter-causal dependence* (ICD) between simultaneously occurring causes [3], the NOR model is no longer valid and can lead to prediction errors, i.e., overestimation or underestimation, when compared to the true probability distribution for the multi-causal effect [8, 9, 10]. The ICD information can be obtained from data reflecting the effects of multiple causes. For the case study discussed in this paper, we are using numerical data derived from simulations carried out using industry-standard physics-based simulation tools. We can do this because of the nature of the problem that we are studying. The advantage of this approach is that the results are more robust than expert opinion, and reproducible. Based on the classification provided in [11], ICD between simultaneous causes is described in terms of two types of interactions: *positive causality* and *inhibition*. The former is a condition when additional causes always increase the probability of achieving an effect, whereas the latter corresponds to the case when positive causality condition is not true.

To increase the CPT accuracy of a common-effect node variable, the recursive-NOR (RNOR) rule was proposed in [11]. In the RNOR model, available probabilities corresponding to positive causality were included as an extension to the NOR model. However, probability values estimated using the RNOR rule can be invalid when inhibition interactions are involved.

Models that were subsequently proposed to accommodate both inhibition and positive causality include the non-impeding noisy-AND (NIN-AND) tree [12], the inter-causal cancellation model [3], the adaptive recursive NOR (ARNOR) rule [13] and the improved-ARNOR (IARNOR) rule [8]. Additionally, the RNOR rule also suffers from the *asymmetry problem*, which was identified in [3] and has been addressed in [13] by proposing the extended-RNOR (ERNOR) model. Collectively these models are referred to as the *NOR derivatives*.

This paper evaluates the performance of the deterministic NOR derivatives using simulations of the failure of an integrated circuit (IC) under electromagnetic interference (EMI) due to up to ten frequencies as a case study of multi-causal analysis for electrical and electronic systems. In recent studies [8, 9], the comparison made between the probability of IC failures obtained from simulations of two- and three-frequency disturbances and their corresponding NOR estimates has revealed that multi-frequency interactions can be of type synergy, asynergy or inhibition. The IARNOR model proposed in [8] for the prediction of three-frequency failure probabilities based on two-frequency causal dependence information was found to provide more accurate predictions than the NOR, RNOR and ARNOR models. In this paper, the asymmetry problem of the NOR derivatives RNOR, ARNOR and IARNOR, is first resolved by using the ERNOR theory. Furthermore, a new model with increased prediction accuracy (SNOR) is proposed, and validated using data from the simulation-based IC case-study.

The remainder of the paper is organised as follows. Section 2 provides the background and related work on the theory and limitations of the NOR model and its derivatives available in the literature. To overcome those limitations, in Section 3, a new deterministic model has been proposed. This is followed, in Section 4, by a discussion regarding the potential use of deterministic models to overcome the practical limitations of multi-causal analysis for a domain specific case-study. In Section 5, the NOR model is applied to a case-study to identify the variation in the proportion of multi-causal interaction types with respect to increasing number of simultaneous causes. The suitability of the noisy-OR derivatives (including the newly proposed SNOR model) for multi-causal analysis is then evaluated for the case-study in Section 6, which also assesses the prediction accuracy of SNOR for n^{th}-order multi-causal effect probabilities when using available lower-order ICD information (i.e., positive causality and inhibition). The conclusions are provided in Section 7.

2. Background and Related Work

For the completeness of a multi-causal BN (see example in Fig. 1), each node variable in the BN has to be assigned with a CPT, i.e., a probability distribution table conditioned on the Cartesian product (all possible non-repeating combinations) of the states of its parent node variables [14]. Hence, with the increasing number of modelled causes, the CPT entries of the effect node that are required grows exponentially, which is the main limitation that has been focused in this paper. Various existing deterministic functions that are available in the literature, to estimate the missing CPT entries of the effect node in a multi-causal BN have been are discussed below, along with their merits and limitations. Although other limitations (such as the complexity of BN inferences for increasing numbers of causal nodes) are not considered in this paper, they have been discussed elsewhere [14].

2.1. Noisy-OR

Consider a set of k binary-variables, $\mathcal{X} = \{X_1, X_2, ..., X_k\}$, where each variable $X_i \in \mathcal{X}$ represent the presence $(X_i = x_i)$ or absence $(X_i = \bar{x}_i)$ of an individual cause, and a binary-variable E, that corresponds to observing an effect $(E = e)$ or no effect $(E = \bar{e})$. Then, for k causes the CPT table for the effect node E in Fig. 1 will have to be populated with 2^k probability entries of observing the effect, corresponding to the powerset of $\mathcal{X}$, denoted as $\mathbb{Y}$. Using the ICI assumptions of the NOR model [2], the deterministic function to estimate the conditional probability of observing an effect for a given subset of causes $\mathbf{X} \subseteq \mathcal{X}$ that are assumed to be present is given by:

$$\mathcal{N}(e|\,\mathbf{X}) = 1 - \{(1-\lambda))\prod_{i=1}^{n}[1 - \mathcal{P}(e|\,x_i)]\} \tag{1}$$

where n is an integer equal to the number of simultaneous causes (i.e., the size of $\mathbf{X}$), $\mathcal{P}$ denotes probability values obtained from the data, $\mathcal{P}(e|\,x_i)$

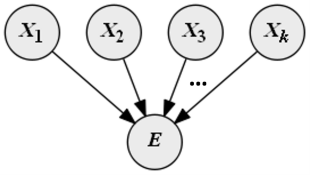

Figure 1: Multi-causal BN example.

is the individual probability of each cause $X_i \in \mathcal{X}$ leading to the effect and λ is the *leak probability* [15], which corresponds to the probability of observing the effect when all modelled causes in the BN are absent. The NOR estimated probability values are denoted using $\mathcal{N}$. For the rest of the paper, the conditional probability of observing an effect given the set $\mathbf{X}$ of simultaneous causes, i.e., $\mathcal{N}(e|\ \mathbf{X})$ is denoted using the short-hand notation $\mathcal{N}_{\mathbf{X}}$ (and similarly for other models).

Merits and Limitations: With the NOR model, only k CPT entries (corresponding to the probability of observing the effect due to individual causes, $\mathcal{P}(e|\ x_i)$, or simply $\mathcal{P}_{x_i}$, $i = 1$ to k) are required to estimate the remaining $2^k - k - 1$ CPT entries of the effect node (note that 1 entry corresponds to λ). Hence, the CPT elicitation is reduced from an exponential to a linear problem.

In addition to the k CPT entries $\mathcal{P}_{x_i}$, which the NOR model requires to complete the CPT entries, the data corresponding to one or more CPT entries of multi-causal effect $\mathcal{P}_{\mathbf{X}}$, where $\mathbf{X} \in \mathbb{Y} \setminus \{x_i\}$ with '$\setminus$' denoting set subtraction, may also be available. However, the available information of those CPT values have no influence on the NOR estimates.

Since the NOR model estimates the probability of multi-causal effect based on ICI assumptions, any difference between the multi-causal probability $\mathcal{P}_{\mathbf{X}}$ and the corresponding NOR estimate $\mathcal{N}_{\mathbf{X}}$ indicates the existence of ICD between the members of the set $\mathbf{X}$ of causes contributing to the effect. Note that, using NOR model in cases of positive causality type interaction leads to underestimation of the probability of multi-causal effect. On the other hand, inhibition indicates overestimation when using the NOR model.

2.2. Recursive Noisy-OR

In 2004, the RNOR model was proposed in [11] as an extension of the NOR model, in which any available CPT value of multi-causal effect, where $\mathcal{P}_{\mathbf{X}}$ is of type positive causality (i.e., $\mathcal{P}_{\mathbf{X}} \geq \mathcal{P}_{\mathbf{x}}$, $\mathbf{x} \subset \mathbf{X}$) can be used in addition to the k individual causal-effect probabilities available, to determine the missing CPT entries.

For $n > 2$, the RNOR rule is given by:

$$\mathcal{R}_{\mathbf{X}} = \begin{cases} \mathcal{P}_{\mathbf{X}}, \\ \text{if available and } \mathbf{X}\text{: positive causality} \\ 1 - \prod_{j=0}^{n-1} \left[\dfrac{1 - \mathcal{R}_{\mathbf{X}\setminus\{x_{j+1}\}}}{1 - \mathcal{R}_{\mathbf{X}\setminus\{x_{j+1}, x_{\mathrm{mod}\ (j+1,n)+1}\})}} \right], \\ \text{otherwise} \end{cases} \tag{2}$$

where $\mathrm{mod}(a, b)$ denotes the modulo operation on integers a and b, '$\mathbf{X}\setminus$' denotes set-subtraction from $\mathbf{X}$, and $\mathcal{R}_{\mathbf{X}}$ indicates the probabilities that are estimated using the RNOR. Use of the modulo function requires an index j running from 0 in the product, but the resulting indices are incremented by 1 to maintain consistency with the cause numbering $\{X_1, .., X_n\}$. Note that, the RNOR rule simply reduces to the NOR if no additional information is provided or if $n = 2$.

Limitations: The RNOR rule has two limitations. First, the $\mathcal{R}_{\mathbf{X}}$ estimates can be out of range $[0, 1]$ if the inter-causal dependence information is of type inhibition, thus making it invalid according to probability theory. The second limit is due to the *asymmetry problem* of the RNOR model discussed in [3, 13]. According to the RNOR deterministic function given in (2), to predict the probability of an effect due to n causes, the probability values corresponding to non-repeating combinations of $(n-1)$ causes in the numerator and $(n-2)$ causes in the denominator have to be used. For example, with the assumption that the probability values corresponding to the lower-order combination of causes $(n-1, n-2)$ are available, the RNOR function for any four causes, $\mathbf{X} = \{x_1, x_2, x_3, x_4\}$ using (2) is given as:

$$\mathcal{R}_{x_1, x_2, x_3, x_4} = 1 - \left[\frac{(1 - \mathcal{P}_{x_1, x_2, x_3})(1 - \mathcal{P}_{x_1, x_3, x_4})(1 - \mathcal{P}_{x_1, x_2, x_4})(1 - \mathcal{P}_{x_1, x_2, x_3})}{(1 - \mathcal{P}_{x_3, x_4})(1 - \mathcal{P}_{x_1, x_4})(1 - \mathcal{P}_{x_1, x_2,})(1 - \mathcal{P}_{x_2, x_3})} \right] \tag{3}$$

For n causes, the total number of all possible non-repeating combinations of these causes, denoted by C_r^n, is given by:

$$C_r^n = \frac{n!}{r!(n-r)!} \tag{4}$$

where $m!$ denotes the factorial of an integer m.

In the example shown in (3), where $n = 4$, the number of non-repeating combinations, C_r^n (where r is an integer denoting the number of lower-order combinations of n) for any two of the four causes is $C_{r=n-2}^n = 6$. Notice that the two of those six possible combinations, i.e., $\{x_1, x_3\}$ and $\{x_2, x_4\}$ are missing in the denominator of (3). Since the number of terms in the numerator and denominator of (2) is always equal, for any integer $n > 2$, the number of combinations that will not be included is equal to $C_{r=n-2}^n - C_{r=n-1}^n$. The asymmetry problem is because of the inconsistencies of RNOR estimates arising due to the fact that the combinations that are not included in (2) purely depend on the arrangement of causes in $\mathbf{X}$. In the previous example, if the arrangement of $\mathbf{X} = \{x_2, x_1, x_3, x_4\}$, then the missing combinations would have been $\{x_2, x_3\}, \{x_1, x_4\}$ instead of the previously missing terms $\{x_1, x_3\}$ and $\{x_2, x_4\}$. So, unless the probabilities corresponding to all non-repeating combinations of lower-order causes ($r = n - 2$) are equal, the RNOR estimates will be inconsistent. For $n = 3$, C_1^3 - $C_2^3 = 0$, which means there will be no missing combinations. Hence, the asymmetry problem of RNOR rule exists for values of $n > 3$.

2.3. Adaptive-RNOR

In [16], the RNOR rule was validated for the application of asthma prediction. In this work, the inhibition type interactions are replaced with corresponding NOR estimates when the RNOR predictions including the inhibition information were found to be invalid. This simple approach is called the *adaptive-RNOR* (ARNOR) rule, which is given by:

$$\mathcal{A}_{\mathbf{X}} = \begin{cases} \mathcal{P}_{\mathbf{X}}, \\ \text{if available and } \mathbf{X}\text{: positive causality} \\ \mathcal{N}_{\mathbf{X}}, \\ \text{if } \mathcal{P}_{\mathbf{X}} \text{ is available; } \mathbf{X}\text{: inhibition and } 0 > \mathcal{R}_{\mathbf{X}} > 1 \\ \mathcal{R}_{\mathbf{X}}, \\ \text{otherwise} \end{cases} \tag{5}$$

Limitation: The ARNOR-rule also shares the asymmetry problem of the RNOR rule.

2.4. Improved-ARNOR

As previously discussed, the RNOR function in (2) considers the ICD information of type positive causality, thus omitting valuable information

on inhibition and potentially resulting in overestimation of the probability of observing the effect. To avoid this, a correction factor was used in the improved-ARNOR (IARNOR) model proposed in [8], which is based on those values of a *degree of synergy* (DoS) metric that correspond to inhibition type interactions. The DoS introduced in [8] is based on the NOR model, and has been found to be useful as it provides:

- a means to identify the existence of ICD, thereby indicating where ICI assumptions are inappropriate;
- a basis for distinguishing between the synergy, asynergy and inhibition interactions;
- a foundation for deriving the correction factors used for the IARNOR model and a further NOR derivative proposed in this paper.

The DoS is obtained from:

$$DoS_{\mathbf{X}} = \frac{\mathcal{P}_{\mathbf{X}} - \mathcal{N}_{\mathbf{X}}}{\mathcal{N}_{\mathbf{X}}} \tag{6}$$

Any non-zero DoS indicates the existence of ICD. More specifically, a positive $DoS_{\mathbf{X}}$ indicates *synergy* between the causal mechanisms, which means the probability of multiple causes leading to an effect is greater than the prediction made with the NOR model. The ICD resulting in a negative $DoS_{\mathbf{X}}$, indicates two types of interactions: *asynergy*, if interactions satisfy positive causality i.e., $\mathcal{P}_{\mathbf{X}} \geq \mathcal{P}(e|\ \mathbf{x})$, where $\mathbf{x} \subset \mathbf{X}$, *inhibition*, otherwise. It should be noted that asynergy and synergy type interactions belong to positive causality, as in both cases the probability of multi-causal effects is greater than the probability of observing the effect due to all lower-order combinations of causes.

Clearly, a negative $DoS_{\mathbf{X}}$ indicates that the NOR estimates will lead to overestimation. Hence, a correction that can reduce the overestimation of the NOR model would improve the accuracy of the ARNOR model. The correction is therefore assumed to be of the form $\mathcal{N}_{\mathbf{X}}(1 + W_{\mathbf{X}})$ where the correction factor $W_{\mathbf{X}}$ is negative. The IARNOR rule using the correction

factor $W_{\mathbf{X}}$ is then:

$$\mathcal{I}_{\mathbf{X}} = \begin{cases} \mathcal{P}_{\mathbf{X}}, \\ \text{if available and } \mathbf{X}\text{: positive causality} \\ \mathcal{N}_{\mathbf{X}}(1 + W_{\mathbf{X}}), \\ \text{if } \mathcal{P}_{\mathbf{X}} \text{ is available; } \mathbf{X}\text{: inhibition and } 0 > \mathcal{R}_{\mathbf{X}} > 1 \\ \mathcal{R}_{\mathbf{X}}, \\ \text{otherwise.} \end{cases} \tag{7}$$

In (7), $\mathcal{I}_{\mathbf{X}}$ denotes the probability of multi-causal effect estimated using the IARNOR function. The correction factor, $W_{\mathbf{X}}$ adds the $DoS_{\mathbf{X}\setminus\{C_1^n\}_j}$ metric corresponding to j unique $(n-1)$-order combinations of type inhibition and divides the sum by the total number of lower-order combinations $\{C_{n-1}^n\}$ to get an average inhibition value as the correction factor in range [-1, 0]. If the probability of observing the effect due to any of $(n-1)$-order combinations, $\mathcal{P}_{\mathbf{X}\setminus\{C_1^n\}_j}$ is unavailable to calculate $DoS_{\mathbf{X}\setminus\{C_1^n\}_j}$, then $\mathcal{P}_{\mathbf{X}\setminus\{C_1^n\}_j}$ is replaced recursively with $\mathcal{I}_{\mathbf{X}\setminus\{C_1^n\}_j}$ in (6). The correction factor is expressed as:

$$W_{\mathbf{X}} = \begin{cases} \frac{1}{C_{n-1}^n} \sum_{j=1}^{C_{n-1}^n} DoS_{\mathbf{X}\setminus\{C_1^n\}_j}, \\ \text{ if } \mathcal{P}_{\mathbf{X}\setminus\{C_1^n\}_j} \text{is available and } \mathbf{X} \setminus \{C_1^n\}_j\text{: inhibition} \\ \frac{1}{C_{n-1}^n} \sum_{j=1}^{C_{n-1}^n} \frac{\mathcal{I}_{\mathbf{X}\setminus\{C_1^n\}_j} - \mathcal{N}_{\mathbf{X}\setminus\{C_1^n\}_j}}{\mathcal{N}_{\mathbf{X}\setminus\{C_1^n\}_j}}, \\ \text{otherwise} \end{cases} \tag{8}$$

In the above expression, the upper-limit of the summation, C_{n-1}^n corresponds to the number of all non-repeating combinations of $n-1$ simultaneous causes in $\mathbf{X}$, and for every j^{th} term of the sum '$\setminus\{C_1^n\}_j$' corresponds to the set subtraction of a unique cause from $\mathbf{X}$. Note that the summation in (8) always reduces to zero for positive causality, thereby including only the inhibition information, which the ARNOR models may omit from multi-causal prediction. The increase in prediction accuracy of the IARNOR model was validated and compared with the existing NOR and ARNOR functions in [8, 10]. Further evaluation of the improved prediction accuracy of the IARNOR model is also provided in Section 6 of this paper.

Limitation: The IARNOR function in (7) inherits the asymmetry prob-

lem of its original parent, i.e., the RNOR model, and hence the application of IARNOR model to construct a CPT for an effect involving more than three simultaneous causes will not be consistent. Since the multi-causal analysis in [8, 17] was limited up to three simultaneous causes, the inconsistency due to the asymmetry problem was not an issue.

2.5. Extended-RNOR

More recently, the asymmetry problem of the RNOR model has been resolved in [13] with the proposal of a new function called the extended-RNOR (ERNOR). This is achieved by including all lower-order combinations with respect to n, i.e., $r = \{n-1, ..., 1\}$. The ERNOR expression [13] is given as:

$$\mathcal{E}_{\mathbf{X}} = \begin{cases} \mathcal{P}_{\mathbf{X}}, \\ \text{if available,} \\ 1 - \left[\dfrac{\prod_{j=1}^{C_{r=n-1}^{n}} (1-\mathcal{E}_{\mathbf{X}\setminus\{C_1^n\}_j}) \times \prod_{j=1}^{C_{r=n-3}^{n}} (1-\mathcal{E}_{\mathbf{X}\setminus\{C_3^n\}_j}) \cdots}{\prod_{j=1}^{C_{r=n-2}^{n}} (1-\mathcal{E}_{\mathbf{X}\setminus\{C_2^n\}_j}) \times \prod_{j=1}^{C_{r=n-4}^{n}} (1-\mathcal{E}_{\mathbf{X}\setminus\{C_4^n\}_j}) \cdots} \right] & \text{while } r > 0, \\ \text{otherwise.} \end{cases} \tag{9}$$

For the example discussed in Section 2.2, where $\mathbf{X} = \{x_1, x_2, x_3, x_4\}$ the ERNOR function is given as:

$$\begin{aligned} &\mathcal{E}_{x_1,x_2,x_3,x_4} = \\ &1 - \left[\frac{(1-\mathcal{P}_{x_1,x_2,x_3})(1-\mathcal{P}_{x_1,x_3,x_4})(1-\mathcal{P}_{x_1,x_2,x_4})(1-\mathcal{P}_{x_2,x_3,x_4})}{(1-\mathcal{P}_{x_3,x_4})(1-\mathcal{P}_{x_1,x_4})(1-\mathcal{P}_{x_1,x_2,})(1-\mathcal{P}_{x_2,x_3})(1-\mathcal{P}_{x_1,x_3})(1-\mathcal{P}_{x_2,x_4})} \right] \\ &\qquad \times \left[(1-\mathcal{P}_{x_1})(1-\mathcal{P}_{x_2})(1-\mathcal{P}_{x_3})(1-\mathcal{P}_{x_4}) \right] \end{aligned} \tag{10}$$

in which all the terms that were missing in the RNOR function in (3) are now included.

Limitations: Although the asymmetry problem is resolved, the ERNOR model does not make use of ICD information of type inhibition.

2.6. Other Models

For the application of multi-frequency immunity analysis discussed in the next section, the above-detailed deterministic functions have been considered

to obtain the conditional probability of observing the failures due to simultaneous disturbances. Although other models that rely on graphical structures for capturing multi-causal interactions are described the literature (e.g., [18, 19, 12]), these are not considered to be suitable for the multi-frequency analysis of this work. For instance, the NIN-AND model proposed by [18] is a tree that is constructed using the noisy-AND gates to include both positive causality and inhibition type interactions. However, as argued in [12], it is error-prone and non-trivial to construct the tree prior to the analysis.

In [19, 20, 21], causal nodes that inhibit and enhance the effect are grouped separately when constructing the BN structure. However, it is not possible to follow this grouping approach for applications such as multi-frequency analysis, where an individual cause can have different causal interactions depending on other simultaneously occurring cause(s).

The inter-causal cancelling model proposed in [12] is limited to causes that inhibit each other. Additionally, when there is no awareness of the causal mechanism(s) between simultaneously occurring causes, which is the case in many domains, the CPT entries required for the inter-causal cancellation model are usually unavailable.

There are other generalizations of the NOR models as mentioned in [11], e.g., the noisy-MAX [15] or the generalized NOR (GNOR) [22], which can consider variables with multiple states. However, since this paper is concerned only with binary variables those models are not considered here.

2.7. Summary of NOR Derivatives

To increase the accuracy of the CPT entries of the effect node in a multi-causal BN (as shown in Fig. 1), all existing NOR derivatives discussed in Section 2 use ICD information. However, as summarized in Table 1, the NOR model and all of its existing derivatives have limitations; either they do not make use of all available information or they fail to provide consistent CPT entries due to the asymmetry problem. The ERNOR model overcomes the asymmetry problem of the RNOR model (and its derivatives, ARNOR and IARNOR) for $n > 3$. However, the ERNOR model can lead to invalid predictions for ICD of inhibition type interactions. A new model, called the *super-noisy-OR* (SNOR) model, is therefore proposed in the next Section to overcome the limitations of the NOR model and its existing derivatives.

Table 1: Summary of advantages and limitations of the NOR model and its derivatives.

Model	Advantages	Limitations
NOR	- Expert elicitation in multi-causal BN reduced to linear problem with ICI assumptions.	- Omits any available information on inter-causal dependence for multi-causal effect predictions.
RNOR	- Improves CPT accuracy using ICD information of type positive causality.	- Invalid for inhibition type interactions. - Inconsistent multi-causal effect prediction (for n $\geq$ 4), due to the asymmetry problem.
ARNOR	- Avoids invalid predictions of RNOR model.	- Omits inhibition knowledge when RNOR model results are invalid. - Asymmetry problem inherited from RNOR model.
IARNOR	- Includes inhibition knowledge using a correction factor.	- Asymmetry problem inherited from RNOR model.
ERNOR	- Resolves asymmetry problem of RNOR function.	- Invalid for inhibition type interactions.
SNOR	-Does not suffer from asymmetry problem. -Improves CPT accuracy using all types of ICD information.	- The CPT accuracy depends on the order of available ICD information.

3. Super-NOR (SNOR) Model

The NOR prediction of multi-causal effect, $\mathcal{N}_{\mathbf{X}}$ is based on ICI assumptions and does not use available ICD information on lower-order combinations of individual causes in $\mathbf{X}$ (i.e., $\mathbf{X} \setminus \{C_r^n\}$, where $r = 2$ to $n-1$). Hence, the SNOR model is proposed to include both positive causality and inhibition type ICD information available using two correction factors $Y_{\mathbf{X}}$ and $Z_{\mathbf{X}}$ on DoS metrics of $(n-1)$-order combinations of $\mathbf{X}$. The correction factor $Y_{\mathbf{X}}$ and $Z_{\mathbf{X}}$ are used to include positive DoS metrics corresponding to synergy type interactions and negative DoS metrics corresponding to asynergy and inhibition type interactions, respectively. The correction is therefore assumed to be of the form $\mathcal{N}_{\mathbf{X}}(1+Y_{\mathbf{X}})(1+Z_{\mathbf{X}})$, where $Y_{\mathbf{X}}$ is positive real number, $\mathbb{R}$ and $Z_{\mathbf{X}}$ is negative number in the range [-1,0]. Using $Y_{\mathbf{X}}$ to refine NOR estimate can increase the probability value greater than one due to synergy type interactions, in such cases the SNOR estimates that lead to values greater than 1 is considered as 1.0. The SNOR equation is given as:

$$\mathcal{S}_{\mathbf{X}} = \begin{cases} 1.0, \text{ if } \mathcal{N}_{\mathbf{X}}(1+Y_{\mathbf{X}})(1+Z_{\mathbf{X}}) > 1 \\ \mathcal{N}_{\mathbf{X}}(1+Y_{\mathbf{X}})(1+Z_{\mathbf{X}}), \text{ otherwise.} \end{cases} \tag{11}$$

The correction factors in (11) are expressed as:

$$Y_{\mathbf{X}} = \begin{cases} \frac{1}{C_{n-1}^{n}} \sum_{j=1}^{C_{n-1}^{n}} \{DoS_{\mathbf{X}\setminus\{C_1^n\}_j} > 0\}, \text{ if } \mathcal{P}_{\mathbf{X}\setminus\{C_1^n\}_j} \text{is available} \\ \frac{1}{C_{n-1}^{n}} \sum_{j=1}^{C_{n-1}^{n}} \{\frac{\mathcal{S}_{\mathbf{X}\setminus\{C_1^n\}_j} - \mathcal{N}_{\mathbf{X}\setminus\{C_1^n\}_j}}{\mathcal{N}_{\mathbf{X}\setminus\{C_1^n\}_j}} > 0\}, \text{otherwise} \end{cases} \tag{12}$$

$$Z_{\mathbf{X}} = \begin{cases} \frac{1}{C_{n-1}^{n}} \sum_{j=1}^{C_{n-1}^{n}} \{DoS_{\mathbf{X}\setminus\{C_1^n\}_j} < 0\}, \text{ if } \mathcal{P}_{\mathbf{X}\setminus\{C_1^n\}_j} \text{is available} \\ \frac{1}{C_{n-1}^{n}} \sum_{j=1}^{C_{n-1}^{n}} \{\frac{\mathcal{S}_{\mathbf{X}\setminus\{C_1^n\}_j} - \mathcal{N}_{\mathbf{X}\setminus\{C_1^n\}_j}}{\mathcal{N}_{\mathbf{X}\setminus\{C_1^n\}_j}} < 0\}, \text{otherwise} \end{cases} \tag{13}$$

Similar to the IARNOR correction factor, the SNOR correction factors are recursive when the probability of any $(n-1)$-order combination, $\mathcal{P}_{\mathbf{X}\setminus\{C_1^n\}_j}$ is not available to calculate $DoS_{\mathbf{X}\setminus\{C_1^n\}_j}$.

3.1. Properties of SNOR

Three main properties of the SNOR model in (11) include:

- if $\mathbf{X}_\kappa = \varnothing$ the SNOR reduces to NOR, i.e., in the absence of ICD information $Y_{\mathbf{X}}, Z_{\mathbf{X}} = 0$, such that $\mathcal{S}_{\mathbf{X}} = \mathcal{N}_{\mathbf{X}}$;
- the SNOR probability estimates are always valid i.e., in range [0, 1] regardless of the type of ICD information provided;
- the SNOR probability estimates are independent of the number of causes (n) in $\mathbf{X}$ because it takes account of the ICD information of all possible lower-order $(n-1)$ combinations, hence does not suffer from the asymmetry problem.

Because of the above properties, the SNOR model overcomes the limitations of the NOR model and its derivatives, as outlined in Table 1.

4. Case Study: Multi-causal Analysis for Electromagnetic Interference

All electrical and electronic systems give rise to electromagnetic disturbances that have the potential to disrupt the operation of the system itself and/or other electrical and electronic systems in its operating environment. Such electromagnetic interference (EMI) can occur at frequencies that range from a few Hz up to many GHz, with the result that multiple frequencies may contribute to real-world EMI-related system failures. The conventional approach is to investigate EMI phenomena using simple single frequency (i.e., *single cause*) analysis. However, as electronic systems are increasingly used to provide safety-related functions there is increasing interest in analysing the impact of more realistic multi-frequency (i.e., *multi-causal*) EMI threats.

4.1. Practical Considerations

Depending on the frequencies, relative phases and amplitudes of the contributing electromagnetic disturbances, there is an infinitely large number of possible EMI waveforms that could be considered. Recent studies involving both simulations [9] and experimental measurements [10] of multi-frequency EMI have demonstrated the possibility of both positive causality (synergy and asynergy) and inhibition type interactions in multi-frequency EMI causing IC failures. Positive causality may mean that ICs that pass a single frequency test may still fail under multi-frequency EMI, whereas inhibition may mean that ICs are less likely to fail under multi-frequency EMI than for single frequency EMI, which could lead to over-engineering. Hence, the significance of multi-causal EMI analysis is very clear for all systems that must operate in a multi-frequency environment.

With respect to the number of frequencies considered, single frequency EMI testing is a linear problem in terms of cost and measurement time, whereas multi-frequency testing (by analogy with the application of BN for multi-causal analysis shown in Fig. 1) is an exponential problem. For instance, if n different frequencies are considered as the set of causes, then all possible non-repeating combinations of r simultaneous frequencies, i.e., $\sum {}_nC_r$, for $r = 2$ to n, would require $2^n - n - 1$ additional immunity tests, making multi-frequency EMI testing economically impracticable. A possible way to limit total number of tests required for multi-frequency analysis without compromising the number of frequencies analysed could be to predict the impact of untested frequency combinations from the tested ones. The

deterministic functions discussed in Section 2 and 3 for the elicitation of CPT entries for the effect node in multi-causal BNs can therefore be used for this purpose.

4.2. Case-Study Description

The case-study considered in this paper is a voltage-controlled ring oscillator (RO) circuit that is intended to generate a sinusoidal output at a frequency of $F_T = 955$ MHz when biased with a constant voltage [23]. For the purposes of this study, the behaviour of the oscillator circuit is simulated with the EMI signals (single or multi-frequency) superimposed on the bias voltage while the oscillator output in monitored for a time period of 700 ns.

At any time-step of 1.057 ns, a relative frequency deviation of $\pm 5\%$ from F_T due to EMI is considered as a failure. The probability of failure is calculated by dividing the number of failures by the total number of time-steps over which the output frequency was recorded. For example, the output frequency recorded during the injection of a single frequency disturbance X_1 at 100 MHz is shown in Fig. 2, where the red dashed lines indicate the acceptable F_T variation of $\pm 5\%$ and blue circles and green triangles indicate failures and non-failures, respectively.

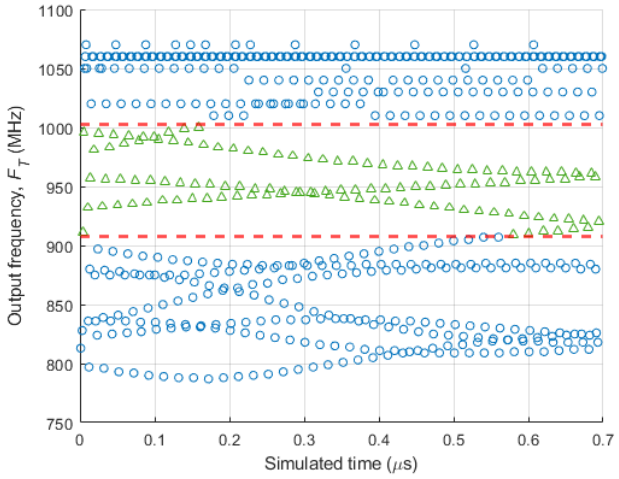

Figure 2: Simulated RO output for single frequency EMI at 100 MHz.

In this example, the probability of failure (effect) due to the presence of a 100 MHz disturbance (single cause, $X_1 = x_1$) is 0.819. As the remaining disturbances (i.e., $X_j = \bar{x}_j$, for $j = 2$ to 10) are absent in this

case, the conditional probability of failure for this example is expressed as $\mathcal{P}(E = e|\ \mathcal{X} = \{x_1, \bar{x_2}, \bar{x_3}, \ldots, \bar{x_{10}}\})$, or more simply as $\mathcal{P}(e|\ \mathbf{X} = \{x_1\})$. Applying the failure criterion to simulation results corresponding to all other single frequency disturbances and possible combinations of multi-frequency disturbances, a complete CPT for observing the failure was obtained. Due to space constraints, it is not possible to include the CPT in this paper.

5. Analysis of Multi-causal EMI

The probability of failure due to all possible combinations of single and multi-frequency disturbances is available from the simulated case study data. In addition, the NOR function (1) is used to estimate the probability of each multi-frequency combination using the single frequency probability values given in Table 2. To avoid calculations using existing NOR derivatives leading to invalid CPD entries [11], any probability values greater than 0.9999 and less than 0.0001 are approximated as 0.9999 and 0.0001, respectively (and displayed as 1.0 and 0.0). This means that *complete failure* and *no failure* are indicated by 1.0 and 0.0, respectively. In the absence of EMI, the RO is assumed to have no failures, hence the leak probability, λ in the NOR function, is 0.0 for this case study.

Table 2: Probability of failure due to single frequency EMI

Frequency (MHz)	X_1 (100)	X_2 (200)	X_3 (300)	X_4 (400)	X_5 (500)	X_6 (600)	X_7 (700)	X_8 (800)	X_9 (900)	X_{10} (1000)
$\mathcal{P}_{\mathbf{X}}$	0.819	0.813	0.788	0.980	0.977	1.0	1.0	0.178	0.064	0.002

5.1. Identification of Inter-Causal Dependence

The estimated *DoS* values and interaction types for all combinations of 2 to 9 causes are shown in Figs. 3 and 4. From Figs. 3 and 4 it can be seen that the *DoS* values associated with some frequency combinations are relatively large. For example, $DoS\ (\mathbf{X} = \{x_8, x_9\}) = 120\%$ and $DoS\ (\mathbf{X} = \{x_2, x_8, x_9\}) = -94.65\%$. Such large *DoS* values indicate strong ICD, leading to significant prediction errors when using the ICI assumptions of the NOR function (which are clearly invalid for these cases).

The pie charts in Fig. 5 depict the variation in the proportion of causal interaction types as the number of simultaneous frequencies is increased. It can

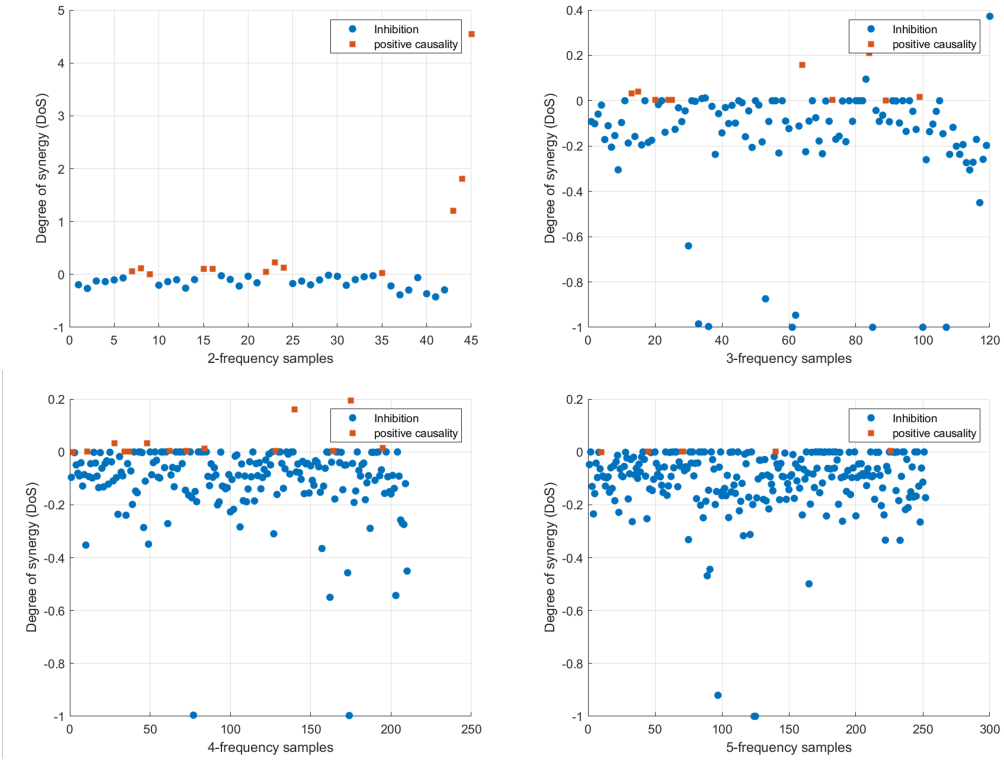

Figure 3: *DoS* of multi-causal EMI generated from 2 to 5 frequency combinations.

be seen that more than 70% of the ICD for all higher-order multi-frequency disturbances in the case study data are of type inhibition. Another useful observation is the proportion of positive causality type interactions, which reduces from 27% for 2-frequency combinations to zero for 6-frequency and higher combinations. This means that the use of NOR function in multi-frequency analysis will mostly overestimate the failure probabilities e.g., the probability of failure corresponding to the 10-frequency combination was calculated from the data as $\mathcal{P}_{\mathbf{X}} = 0.9518$ and the NOR estimate from (1) as $\mathcal{P}_{\mathbf{X}} = 1.0$. Using (6), the $DoS = -0.0482$ and since there are multiple lower-order combinations which have the probability of failure greater than the 10-frequency combination, it is concluded that the ICD type of the 10-frequency combination is inhibition.

The high proportion of inhibition observed with increasing number of frequencies indicates that the NOR estimates can provide a safety-margin

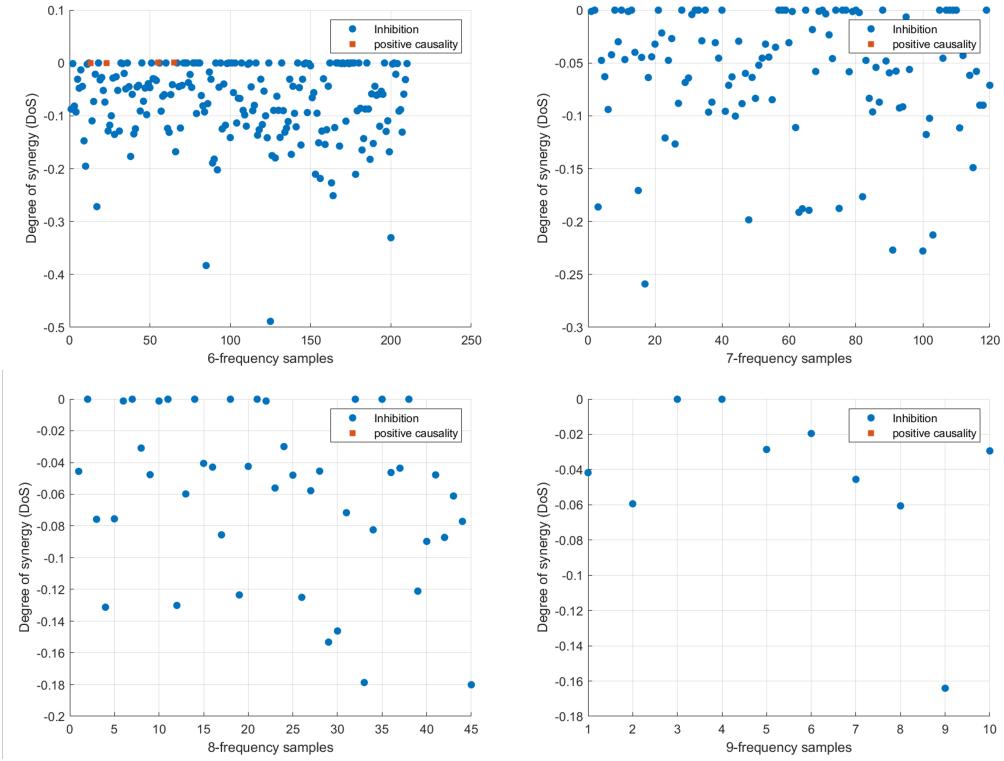

Figure 4: *DoS* of multi-causal EMI generated from 6 to 9 frequency combinations.

when predicting the multi-frequency failure probabilities, on the positive side, but could also lead to over-engineering, on the negative side.

6. Evaluation of the Noisy-OR and its Derivatives

Assuming $\mathbb{X}$ as the powerset of $\mathbf{X}$ (with n causes) for which the multi-causal effect probability is to be calculated, the multi-causal effect probabilities of any $\kappa \leq 2^n - (n+1)$ subset(s) of $\mathbb{X} \setminus \{\varnothing, \mathbf{X}, \{x_i\} \in \mathbf{X}\}$ are denoted as $\mathbb{X}_\kappa$. Including the n individual cause-effect probabilities, all the known probabilities are collectively denoted as $\mathbb{X}_{\kappa,n}$.

For example, if $\mathbf{X} = \{x_1, x_2, x_3\}$, then $\mathbb{X} = \{\{\varnothing\}, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}\}$. Assuming all second-order combinations of $\mathbf{X}$ ($\kappa = C_{r=2}^{n=3} = 3$) are available, in addition to the $n = 3$ individual cause-effect probabilities, $\mathcal{P}_{\{x_i\}}$, for $i = 1$ to 3, the *available CPT information* is $\mathbb{X}_{\kappa,n} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$.

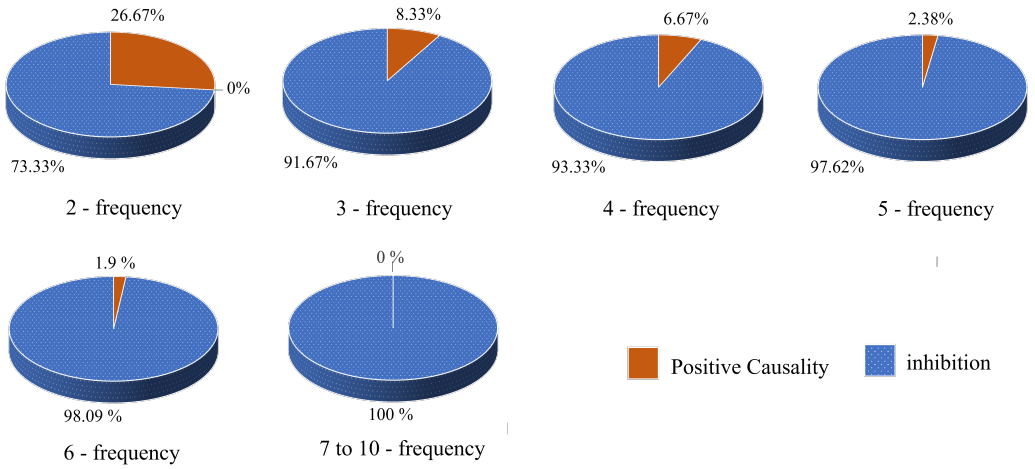

Figure 5: Change in the proportion of multi-causal interaction type with increasing number of simultaneous disturbances.

For the case-study, the failure probabilities corresponding to all the 1013 multi-frequency combinations is available from data. Hence, in Section 6.2, for assumptions of $\mathbb{X}_{\kappa,10} = \{\sum_{r=1}^{2} C_r^{10}, \sum_{r=1}^{3} C_r^{10}, ..., \sum_{r=1}^{9} C_r^{10}\}$, the evaluation of the NOR and its derivative models: RNOR, ARNOR, ERNOR, IARNOR and SNOR, is done by comparing the errors in predicted higher-order multi-frequency failure probabilities with corresponding actual failure probability values obtained from the data. Due to space constrains, only the distribution of errors corresponding to 3-frequency failure predictions is provided in Section 6.2, nonetheless, the mean and the standard deviation of the prediction error distributions corresponding to 3 to 9-frequency combinations are summarized in Tables 3 and 4, respectively. For this evaluation, failure prediction of n^{th}-order frequency combinations is done with the assumption that the failure probability values of all lower-order frequency combinations are known, i.e., $\kappa = 2^n - (n+1)$.

With respect to the complexity of the prediction models, the NOR model is always a linear model, i.e., it uses only the probability entries corresponding to the single-cause effect to predict multi-causal effects, whereas, the complexity of the NOR derivatives varies depending on the number of multi-causal effect ICD information available. Hence, in Section 6.3, further evaluation of the NOR derivative models is done with the assumption that the failure probability of different sets of lower-order frequency combinations are

available to predict failures due to higher frequency combinations.

6.1. Criteria for Comparing Prediction Capability

Bias and *precision* [24] are two different parameters that used for evaluating the quality of measurement and/or prediction populations. Bias is a measure of the difference between the expected value (mean) of a number of estimates from the true value, whereas, precision is a measure of how similar the population of results are to each other, which reflects confidence in the results. In this work the capability of NOR and its derivative models to make reliable predictions is evaluated based on the *standard deviation* (indicating precision) of the error population corresponding to difference between the EMI failure predictions and measurements for the n-frequency combinations, and the mean value of the error population, Which should ideally be close to zero (the "true" value if the predictions accurately reflect the measurements). These two parameters, however, may present in four possible combinations:

- Precise and biased: small standard deviation of prediction errors but mean value of prediction error population displaced from zero - poor;
- Imprecise and biased: larger standard deviation of prediction errors with mean value of prediction error population displaced from zero - poor;
- Imprecise and unbiased: larger standard deviation of prediction errors but mean value of prediction error population closer to zero - better;
- Precise and unbiased: small standard deviation of prediction errors with mean value of prediction error population close to zero - best.

As shown in Fig. 6 for illustration purpose, the best prediction model would provide results with a distribution that is closely clustered (i.e., precise) around the true value (i.e., having low bias), as illustrated in Fig. 6.a). Models that produce distributions of results that have notable bias are poor, whatever their degree of precision, because the resulting mean value does not adequately reflect the true value (see Figs. 6.c-d). Hence, models that minimise the bias of the distribution are better than the latter, irrespective of their degree of precision (e.g. Fig. 6.b).

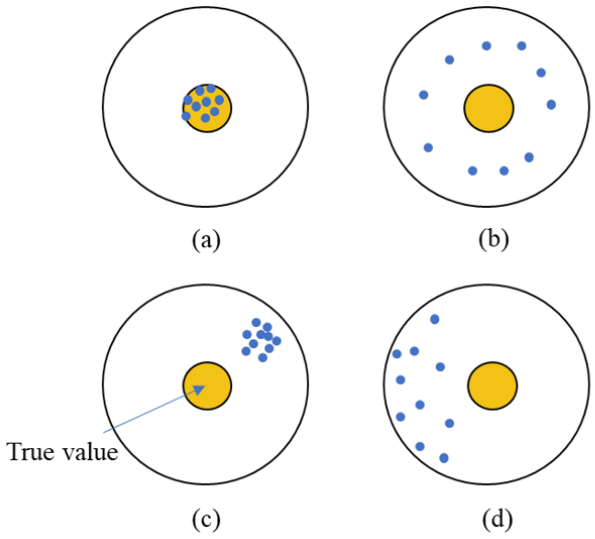

Figure 6: Criteria used to evaluate prediction models: (a) high precision and low bias, (b) low precision and low bias), (c) high precision and high bias (d) low precision and high bias.

6.2. Comparison of Prediction Capability

For $\mathbb{X}_{\kappa,n} = \sum_{r=1}^{2} C_r^{10}$, the distribution of failure prediction errors made by the NOR and its derivatives corresponding to the 3-frequency combinations, C_3^{10} are shown in Figs. 7 and 8, respectively. It should be noted that whereas the NOR model uses only the 10 single-frequency failure probability entries to predict the multi-frequency failures, the NOR derivatives can use any available ICD information on lower-order frequency combinations.

It can be observed from Figures 7 and 8 that, using the NOR model leads to over-estimation of most of the 3-frequency failures when compared to the actual failure probability. On the other hand, in some cases, due to the presence of synergy between 3-frequency combinations the NOR model leads to underestimation of the failure probabilities. The RNOR and ERNOR models (for $n = 3$, both have the same function) use the positive causality type ICD information, i.e., synergy between the 2-frequency combinations to predict a relatively higher probability values when compared to the NOR model. A comparison of distributions corresponding to the 3-frequency failure predictions made by the NOR and the RNOR/ERNOR models from Figs. 7 and 8.a, respectively, show that the number of negative prediction errors can be relatively reduced when RNOR or the ERNOR models are used instead of

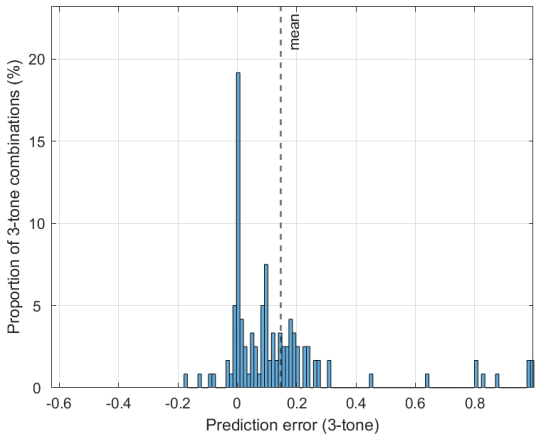

Figure 7: Distribution of failure probability errors for 3-frequency combinations - NOR: $\mu_{\mathcal{N}} = 0.1476; \sigma_{\mathcal{N}} = 0.2335$.

the NOR model.

Due to ICI assumptions, the NOR predictions are always of type positive causality [11]. Hence, the NOR estimates for any multi-frequency combination involving at least one frequency with a probability of EMI failure close to 1.0 (e.g., X_4 to X_7 in Table 2) would lead to almost zero prediction errors for cases involving complete failures. That is the main reason for observing the high peaks at zero in the distributions corresponding to NOR in Fig. 5. Similarly, the RNOR and ERNOR models consider only ICD information of type positive causality to predict a relatively higher probability of failure than the NOR models, hence, a relatively higher peak can be observed at zero for the RNOR and ERNOR distribution shown in Fig. 8.a. For the case-study, the probability of failure predicted by the RNOR and ERNOR models for almost all four and higher-frequency combinations is 1.0, hence the distributions of prediction errors are identical to each other for frequency-order combinations, $n > 3$. Note that RNOR and ERNOR models are equivalent for $n < 3$.

For the case-study, as shown in Fig. 5, the proportion of positive causality type interactions can be observed to reduce to 0% with the increase in the number of frequency combinations, thereby increasing the inhibition type information. The RNOR and ERNOR models do not consider inhibition type information even it is known, to avoid invalid predictions. However, the ARNOR, IARNOR and SNOR models provide valid predictions for any type

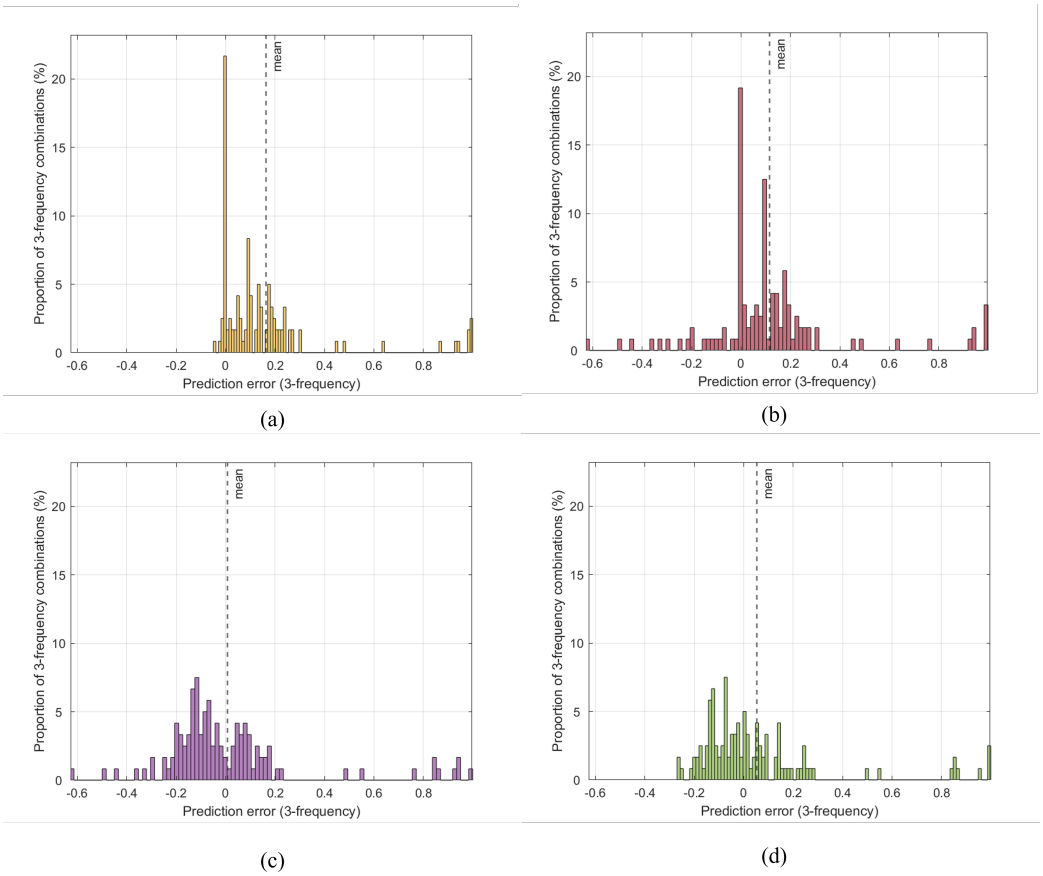

Figure 8: Distribution of failure prediction errors for 3-frequency combinations failure probabilities - (a) RNOR/ERNOR: $\mu_{\mathcal{R}}, \mu_{\mathcal{E}} = 0.1644; \sigma_{\mathcal{R}}, \sigma_{\mathcal{E}} = 0.2414$, (b) ARNOR: $\mu_{\mathcal{A}} = 0.1160; \sigma_{\mathcal{A}} = 0.2813$, (c) IARNOR: $\mu_{\mathcal{I}} = 0.0071; \sigma_{\mathcal{I}} = 0.2863$, (d) SNOR: $\mu_{\mathcal{S}} = 0.0532; \sigma_{\mathcal{S}} = 0.2730$.

information.

A possible drawback of considering inhibition type interactions in addition to positive causality is that the predicted failure values can be underestimated, which in most applications is undesirable compared to overestimation. On the positive side, by considering the inhibition type interactions the mean prediction error can be reduced in general. As observed from the distributions of ARNOR, IARNOR and SNOR shown in Figs 8.b, 8.c and 8.d, respectively, for 3-frequency predictions, a lower bias i.e., a mean prediction error closer to zero, can be achieved when compared to the NOR, RNOR and

ERNOR models (see Tables 3 and 4).

The ARNOR model disregards the possibility of invalid predictions made by the RNOR model when using the inhibition information, by simply replacing any invalid predictions with relevant NOR estimates, and as a consequence excessive underestimated predictions are made by the ARNOR model (see Figs 8.b). The IARNOR model includes the inhibition information discarded by the ARNOR model to avoid invalid predictions using correction factors derived from *DoS* metrics. From Table 3, it can be observed that the mean prediction error of the IARNOR model significantly less than the other models. Finally, the SNOR model, which uses correction factors to include positive causality and inhibition type information has both lower mean prediction error and comparatively less underestimation of failure probabilities than the ARNOR and IARNOR models.

Table 3 provides the standard deviation of failure prediction errors corresponding to 3 to 9-frequency for NOR and its derivatives. It can be observed that the standard deviation of SNOR model predictions is slightly higher than the NOR, RNOR and ERNOR models, however, for most orders it is comparable. However, the standard deviation of ARNOR and IARNOR is higher than the SNOR model.

Table 3: Mean prediction error of n^{th}-order multi-frequency disturbances

Frequency order, n	NOR ($\mu_{\mathcal{N}}$)	RNOR/ERNOR ($\mu_{\mathcal{R}}, \mu_{\mathcal{E}}$)	ARNOR ($\mu_{\mathcal{A}}$)	IARNOR ($\mu_{\mathcal{I}}$)	SNOR ($\mu_{\mathcal{S}}$)
3	0.1476	0.1644	0.1160	0.0071	0.0532
4	0.0961	0.1001	0.0348	-0.0170	-0.0721
5	0.1057	0.1062	0.0510	0.0037	0.0043
6	0.0726	0.0726	0.0282	-0.0398	-0.0353
7	0.0584	0.0584	0.0142	-0.0103	-0.0149
8	0.0597	0.0597	0.0097	-0.0169	0.0013
9	0.0449	0.0449	0.0153	-0.0247	-0.0147

6.3. Comparison of Model Complexity

Due to the ICI assumptions, the NOR model always requires 10 single-frequency failure probability entries to predict multi-frequency failure probabilities. To increase the prediction accuracy the NOR derivatives can additionally use available ICD information i.e., lower-order multi-frequency probability entries. To compare the duality of complexity (in terms of number

Table 4: Standard deviation of n^{th}-order multi-frequency disturbances

Frequency order, n	NOR ($\sigma_{\mathcal{N}}$)	RNOR/ERNOR ($\sigma_{\mathcal{R}}, \sigma_{\mathcal{E}}$)	ARNOR ($\sigma_{\mathcal{A}}$)	IARNOR ($\sigma_{\mathcal{I}}$)	SNOR ($\sigma_{\mathcal{S}}$)
3	0.2335	0.2414	0.2813	0.2863	0.2730
4	0.1345	0.1322	0.2214	0.2395	0.1856
5	0.1301	0.1304	0.2078	0.2121	0.1406
6	0.0738	0.0738	0.1503	0.1504	0.0910
7	0.0602	0.0602	0.1836	0.1834	0.0655
8	0.0516	0.0516	0.1708	0.1665	0.0553
9	0.0469	0.0469	0.0858	0.0692	0.0361

of entries used to make predictions) and the prediction accuracy, the average failure probability of n-frequency combinations obtained from the data, the NOR and its derivatives have been compared for different values of κ, i.e., number of multi-frequency probabilities known. For example, if all the 2-frequency combinations are assumed to be known, then $\kappa = C_2^{10} = 45$.

As shown in Fig. 9, the NOR rule overestimates the average probability of failure when compared to actual data and the predictions converges towards 1.0 indicating that the NOR almost always predicts the effect as complete failure with the increase in the number of frequencies. A similar observation is found for the RNOR and ERNOR models, but the predictions converge much faster towards 1.0 when compared to the NOR model. From Figs. 9—11, it can be noted that increasing the number of multi-frequency probability entries available, i.e., $\kappa = \{C_1^{10}, \sum_{r=2}^{4} C_r^{10}, \sum_{r=2}^{6} C_r^{10}\}$, does not have any impact on the RNOR and ERNOR predicted values. Hence for the case-study, RNOR and ERNOR models do not increase the prediction accuracy even if additional ICD information is provided.

The magnitude of difference between the average failure probability obtained from the actual data and the prediction models that consider inhibition and positive causality type information, i.e., ARNOR, IARNOR and SNOR is much lower compared to the NOR model. The least difference in general was found when using the SNOR model. As shown in Fig. 9, knowing the failure probability entries of 2-frequency combinations, $\kappa = 45$, the SNOR model can predict the average failure probability due to 3 to 9-frequency combinations with reduced difference from the actual values, when compared to NOR and all existing derivatives. With increased complexity,

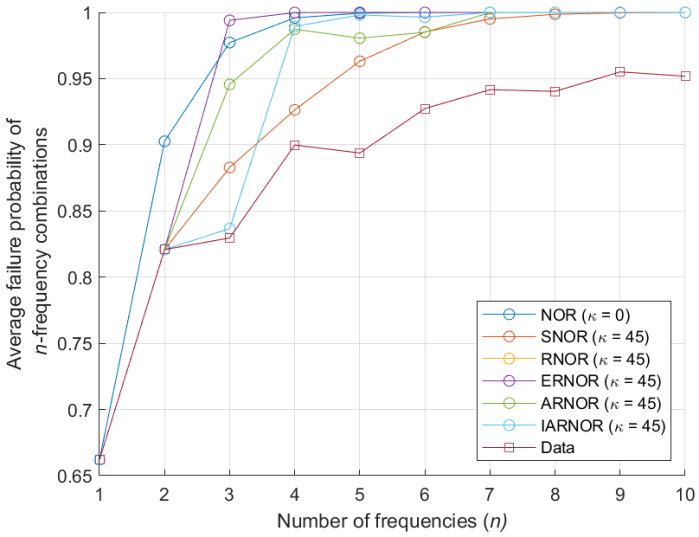

Figure 9: Prediction accuracy of the NOR and its derivatives (with assumption: 10 single and 45 multi-frequency $\kappa = C_2^{10}$ probability values are available) in comparison to the data.

i.e., with the assumption of more available multi-frequency failure probability entries, the SNOR prediction accuracy can be further increased.

7. Conclusions

For the multi-frequency EMI case-study, analysis with NOR facilitates prediction of failure probabilities of higher-order ($n \geq 2$) multi-frequency EMI using ICI assumptions, thus requiring only n single-frequency probability entries to predict 2^n multi-frequency combinations. Based on the results shown in Fig. 9, the average failure probability of n-frequency combinations estimated using the NOR is always higher than the average probability obtained from corresponding data. Note that, the difference between the average probabilities between NOR and data reduces as the order of multi-frequency combinations increases.

With ICI assumptions, the NOR model mostly (and always for $n > 6$) overestimates the probabilities of multi-frequency EMI failures. This is due to the relatively small proportion (or nil) of synergy type interactions and greater prevalence of inhibition type interactions observed in the results from the case-study. If positive causality information is available, then the existing

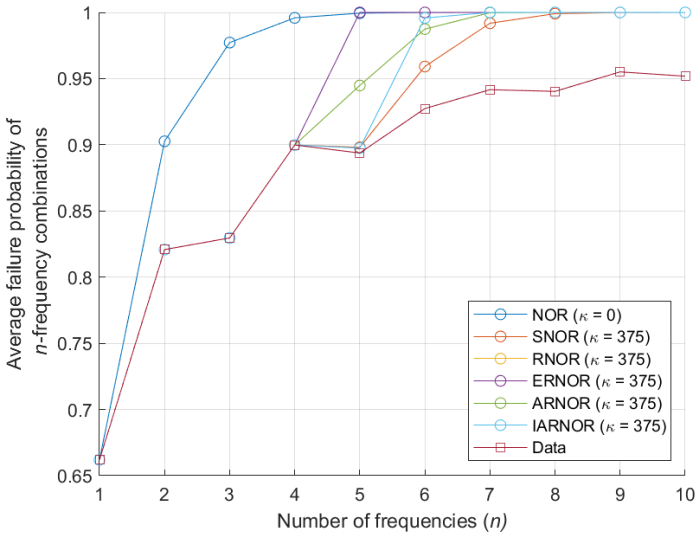

Figure 10: Prediction accuracy of the NOR and its derivatives (with assumption: 10 single and 385 multi-frequency $\kappa = \sum_{r=2}^{3} C_r^{10}$ probability values are available) in comparison to the data.

NOR derivatives (the RNOR and ERNOR models) can be used to predict the multi-causal effect much more reliably than the NOR model. However, the RNOR and ERNOR models cannot be used to include available information on multi-causal interactions of type inhibition. In such cases the ARNOR, IARNOR and SNOR models, which can include any type of ICD information, can be used.

The SNOR model proposed in this paper aims to eliminate the observed limitations of the NOR model and its existing derivatives. Moreover, the superior CPT prediction accuracy of the SNOR model is successfully validated using the case-study based on numerical simulations of EMI-induced failure of a sample IC design when injected with up to ten simultaneous frequencies. Hence, with the availability of ICD information, the SNOR model predicts the n-cause effect probability (for $n > 2$) at a much lower mean error than the NOR model and its derivatives except the IARNOR. However, the magnitude of the error reduction depends on the number of multi-causal probability entries available, and is smaller when the order of the prediction is close to the order of the available data. The SNOR predicted values are equivalent to or converges towards the NOR estimates with an increasing

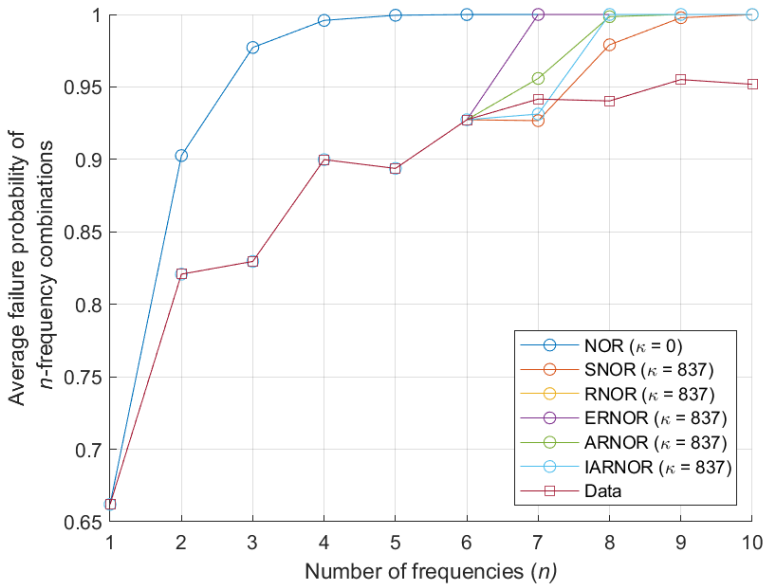

Figure 11: Prediction accuracy of the NOR and its derivatives (with assumption: 10 single and 847 multi-frequency: $\kappa = \sum_{r=2}^{6} C_r^{10}$ probability values are available) in comparison to the data.

difference between the order of ICD information available and the order of multi-causal effect predicted.

In the multi-frequency EMI application, it seems unlikely that all combinations would not be collected in measurements for an achievable number of concurrent frequencies, whereas the number of frequencies that can be concurrently achieved in measurements is restricted by practical limitations. Thus the interest in predicting effects for higher orders of causes from available data on lower orders is because measurements may not be practicable at all orders of interest. However, if the lower-order information is incomplete, requiring missing entries to be predicted with associated error, then an overall accuracy penalty is to be expected. Further work could investigate aspects such as whether the SNOR model may be of benefit in other application domains, or extending the underlying BN to facilitate causal inference.

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